



On the Persistence of Real Interest Rates: New Evidence from Long-Horizon Data

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Abstract

Since the paper of Rose (1988), a large literature has emerged on testing the stationarity of real interest rates using a variety of econometric procedures. In this study, we emphasize that the low power of standard unit root tests, especially with short data spans, may have caused researchers to incorrectly conclude that the rates are nonstationary. We present new unit root test results for four major ex-post and ex-ante real interest rates using monthly long-horizon data spanning the last century. The principal tenet of our analysis is that with the increased power of the tests, we are able to reject the unit root null hypothesis at the 1% level of significance, although shocks impinging upon real interest rates are rather persistent, and the half-life is as much as 7.55 years.

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1 Introduction

The past two decades have seen numerous empirical studies of the relationship between interest rates and expected inflation. According to the Fisher (1930) hypothesis, nominal interest rates should vary one-for-one with expected inflation in the long-run. Thus, the ex-ante real interest rate, which represents the difference between the nominal interest rate and the expected decline in the purchasing power of money, must be stationary if the Fisher relationship is to hold.

However, the stationarity of real interest rate has been questioned in several studies including Rose (1988). Rose (1988) asked the following question: Is the real interest rate stable? Using post-war data from 18 OECD countries and the conventional Dickey and Fuller (1979) unit root test, he failed to reject the presence of a unit root in the real interest rate. This conclusion is reached without analysing the real interest rate; simply finding that the nominal interest rate and inflation have different orders of integration means that any linear combination of the two variables is a nonstationary process with a unit root.

Rose (1988) points out that a unit root in the real interest rate has serious implications not only for the Fisher hypothesis but also for the validity of the consumption-based capital asset pricing model (CCAPM) attributed to Lucas (1978). This model is characterised by a representative agent that maximises periodic utility subject to an intertemporal budget constraint. More importantly, however, the first-order Euler equation derived from this model implies that the time-series properties of the real interest rate (or the real asset return) and the growth rate of consumption should be similar. Given that the latter variable is found not to contain a unit root, nonstationarity of the former represents a violation of the conditions for the CCAPM. Consequently, it seems important to assess if the real interest rate is stationary or if it exhibits unit root behavior.

Since Rose (1988), a great deal of effort has been devoted to verifying the statistical properties of the real interest rate. The two most common approaches used in the literature involve either testing for a unit root in the real interest rate or for cointegration in systems containing inflation and nominal interest rates. In this case, the real interest rate is stationary if the nominal interest rate and inflation are nonstationary and cointegrated. This literature includes MacDonald and Murphy (1989), Mishkin (1992), Wallace and Warner (1993), Crowder and Hoffman (1996), Koustas and Serletis (1999) and Rapach and Weber (2004). The evidence presented thus far is somewhat mixed.

Further, even when the evidence from cointegration tests is supportive of the notion that movements in nominal interest rates reflect movements in expected inflation, changes in the latter generally seem to have less than a one-for-one effect on the former, implying that inflation is non-neutral. Besides, the evidence is sensitive to choice of the periods and countries analysed. The

results on cointegration thus provide not much support for mean-reversion in real interest rates (Lai, 1997).

As for direct tests for a unit root which are mostly based on the conventional Dickey-Fuller tests, they also yield less than supportive evidence for mean-reversion (e.g. Goodwin and Grennes, 1994, and Phylaktis, 1999). A unit root in the real interest rate is confirmed even when using the powerful tests of Hansen (1999) and Romano and Wolf (2001) as documented by Rapach and Wohar (2004). Finally, Lai (1997), Tsay (2000), and Karanasos, Sekioua, and Zeng (2006) explore the potential presence of a fractional unit root in the real interest rate and conclude that it is mean-reverting, though highly persistent.

One possible reason for the inability to find evidence of a long-run relationship between inflation and the nominal interest rate is the low power of conventional statistical tests to reject a false null hypothesis of a unit root in the real interest rate with a data span corresponding to the short length of the samples used. It does not help that the data are often sampled at high or low frequencies, as the power of the tests depends on the span of the data, rather than its frequency (Shiller and Perron, 1985, Perron, 1989, and Pierse and Snell, 1995).

In this paper, we use a response to the low power problem that has received a lot of attention in the literature, especially the literature testing the validity of purchasing power parity (PPP).¹ This approach involves increasing the power of the tests by increasing the length of the sample period. Specifically, we examine the mean-reverting properties of real interest rates by applying tests to monthly data spanning four major economies and 70 to 125 years of data.

Using long samples, it may be possible to establish statistical significance to reject the unit root null hypothesis; however, if the true value of the largest root of an autoregressive (AR) representation of the real interest rate is close to unity, then shocks will be very persistent, and this stationary process may not be significantly different from a true unit root process in the economic sense. As a result, the emphasis in this paper is initially on testing the unit root null hypothesis. If this hypothesis is rejected, then we focus on measuring the economic implications of the real interest rate's behaviour.

¹ Following Frankel (1986, 1990), a number of authors have noted that the tests typically employed during the 1980s to examine the long run stability of the real exchange rate may have very low power to reject a null hypothesis of real exchange rate instability when applied to data for the recent floating rate period alone. The argument is that if the real exchange rate is in fact stable in the sense that it tends to revert towards its mean over long periods of time, then examination of just one real exchange rate over a period of 25 years or so may not yield enough information to be able to detect slow mean reversion towards PPP. In response to this power problem, several studies have employed long horizon data to test long-run PPP (see, e.g., Lothian and Taylor, 1996, 2000, and Taylor, 2002). These studies uncover support for long-run PPP contrary to those using short data. Given this finding, it seems surprising that the literature on the real interest rate has not pursued this response to the power problem.

What market participants and monetary authorities care about is the degree of persistence in the real interest rate and one measure of persistence is the half-life which is defined as the number of periods it takes for deviations to subside permanently below 50% of the initial response to a shock. Further, by means of impulse response analysis, this study analyzes the adjustment dynamics of real interest rates by evaluating both the sample half-life and its estimation accuracy. Since reporting point estimates only does not convey the inevitable imprecision with which the adjustment speed is measured, confidence intervals are also computed (Cheung and Lai, 2000).²

Finally, the Fisher hypothesis is essentially an ex-ante condition. Nonetheless, much of the extant literature tends to assume that expectations are formed rationally and that the difference between the ex-ante and ex-post real interest rates is equal to a stationary, zero mean forecast error term. This assumption allows researchers to use ex-post instead of ex-ante inflation data and circumvent the issue of specifying how expectations are formed.

However, Evans and Lewis (1995) argue that the use of ex-post inflation data can produce substantial small-sample bias in estimates of the Fisher relationship due to the peso problem; rational anticipations of shifts in the inflation rate which do not occur in the sample. This would result in a persistent wedge between expected and realised inflation and may, as a result, create the appearance of permanent shocks to the real interest rate even when none are truly present (Lai, 1997).

To investigate whether our results are sensitive to the measurement of real interest rates, we consider the properties of both ex-ante and ex-post real interest rates. The expected values of inflation used to construct the ex-ante rates are obtained by means of a signal extraction procedure based on a state space model and a Kalman filter.

The remainder of this paper is set as follows. Section 2 describes the analytical Fisher relationship and explains the econometrics of local-to-unity processes. In Section 3 we discuss the data and report the empirical results. The last section concludes.

² It is of course possible to increase the power of the tests with, for instance, panel unit root tests under cross section dependence, SUR-ECM or panel VAR. This is, in fact, the other response to the low power problem followed in the PPP literature. However, rejection of the unit root null in this case may happen only because one of the variables considered is $I(0)$. Hence, if rejection occurs, it may not be informative and certainly it cannot be concluded that this rejection implies evidence supportive of mean reversion. Using long spans of data means that this problem is avoided (Sarno and Taylor, 2003). Nonetheless, there is the potential problem of structural instability when using long data. In this paper, we follow the PPP literature that utilizes long spans of data and assume that the dynamics of the real interest rate are relatively stable over the sample period. In order to employ more powerful tests and long spans of data, we have to assume relatively stable dynamic processes over long periods (Rapach and Wohar, 2002).

2 The Real Interest Rate and Empirical Methodology

The ex-ante Fisher equation is defined as follows:

$$i_t = r_t + \pi_t^e, \quad (1)$$

where i_t is the nominal interest rate, r_t is the real interest rate and π_t^e is the anticipated inflation rate. For the Fisher effect to be valid i_t and π_t^e have to be cointegrated with vector $[1, -1]$ or, equivalently, the real interest rate, r_t , must be a mean-reverting stationary process.

The expected values of inflation are obtained using a signal extraction procedure. This procedure is used to separate unobservable components, state variables or expected values in our case, from an observable variable containing noise. This is achieved through the application of the law of iterated projections by means of the Kalman filter technique. The estimated model is:

$$y_t = \zeta_t + v_t \quad \text{and} \quad \zeta_{t+1} = \zeta_t + \varsigma_t,$$

where ζ_t is a vector of possibly unobserved state variables, and v_t and ς_t are vectors of mean zero Gaussian disturbances (see Valente, 2003 for a recent application). There are other ways of obtaining data on expected inflation, however. For instance, Data Resources Incorporated, the Michigan survey of consumers, and the Livingston Survey all provide data for inflation forecasts. Unfortunately, they do not provide data for the period under examination. In addition, the core inflation rate is sometimes preferred as a proxy for the expected inflation. Unfortunately, we do not have this data either. Recently, St-Amant (1996) and Gottschalk (2001) use structural VARs to derive estimates of expected inflation. Though promising, this approach requires a number of theoretical identifying restrictions.

If equation (1) does indeed hold, i.e. if the real interest rate is found to be stationary, then although this supports the validity of long-run Fisher hypothesis, it offers little information about the speed at which shocks die out. To obtain such information, explicit computation of persistence is needed and the half-life is used to quantify persistence. The point estimate of the half-life alone does not provide a complete description of the persistence of the real interest rate, however. It needs to be supplemented with confidence intervals in order to gauge the precision of the estimates. The construction of intervals for the half-life using ordinary least squares (OLS) poses a number of problems.

First, these intervals are not valid under the unit root null hypothesis and, even if the Fisher effect holds in the long-run, are biased downwards in small samples. However, given the large samples that we are using, the bias is expected to be small. Second, the commonly used estimate of the half-life $H.L = \ln(1/2)/\ln(\alpha)$ is based on the root (α) of an autoregressive model of order one and assumes that shocks decay monotonically, so it may not be appropriate for higher order AR processes.

Consequently, this may lead the researcher to draw the wrong inference about the speed of adjustment. To remedy this, Cheung and Lai (2000) recommend using impulse response analysis. Both the above issues are addressed formally with the median unbiased estimation (MUE) method of Gospodinov (2004).

2.1 Median Unbiased Estimation³

The econometric method employed in this study is based on inverting the likelihood ratio (LR) statistic of the largest root under a sequence of null hypotheses of possible values for the impulse response and the half-life. We start from the following augmented Dickey-Fuller (ADF) regression which includes lagged first differences to account for serial correlation:

$$y_t = \alpha y_{t-1} + \sum_{i=1}^{k-1} \psi_i \Delta y_{t-i} + \varepsilon_t, \quad (2)$$

where α is a measure of the persistence of the series (Andrews and Chen, 1994) and is cast as local-to-unity ($\alpha=1+c/T$ with c fixed as the sample size $T \rightarrow \infty$), $\varphi = (\alpha, \psi')' \in \Xi \subset R^p$, and the maximum likelihood estimator of φ is $\hat{\varphi}$.

Suppose that we are interested in the null hypothesis that the impulse response function at horizon 1, denoted by θ_1 , is 0.5, versus the alternative $\theta_1 \neq 0.5$, then this null (or restriction) can be written as $h(\varphi)=0$, where $h \equiv \theta_1 - 0.5: R^p \rightarrow R$ is a polynomial of degree 1. Let $\tilde{\varphi}$ denote the restricted maximum likelihood estimator and LR_T the likelihood ratio statistic of the null. Gospodinov (2004) shows that the restricted estimator of α converges at a faster rate than the unrestricted estimator and this helps to obtain a consistent estimate of the nuisance parameter c under the imposed restriction (i.e., null hypothesis). Moreover, the restricted estimation provides consistent estimates of the impulse response functions and thus the half-lives.

The restricted LR estimator of equation (2) under the null hypothesis $h(\varphi)=0$ is:

$$LR_T \Rightarrow \left[\int_0^1 J_c^\tau(s) dW(s) \right]^2 / \int_0^1 J_c^\tau(s)^2 ds, \quad (3)$$

where $J_c^\tau(r) = J_c(r) - \int_0^1 J_c(s) ds$, $J_c(r) = \int_0^r \exp[(r-s)c] dW(s)$ is a homogenous Ornstein-Uhlenbeck process and \Rightarrow denotes weak convergence. The limiting theory of LR is dominated by the near nonstationary component and is not affected by the presence of stationary components as measured by the second term in equation (2), $\sum_{i=1}^{k-1} \psi_i \Delta y_{t-i}$.

The employed method has many interesting features. First, contrary to standard asymptotic and bootstrap methods, which have been shown to have poor coverage properties, this method

³ This section draws on Gospodinov (2004). See this paper for further details.

parameterizes α as a function of T and is expected to yield better small-sample and coverage performance. Second, the LR statistic does not require variance estimation for studentization. It is criterion function-based and is tracking closely the profile of the objective function. Also, the inversion of the LR statistic appears to shift the confidence intervals away from the nonstationarity region much more often compared to methods based on inverting the OLS estimator of α such as the grid bootstrap of Hansen (1999). The employed method is also expected to yield tight confidence intervals which make it highly informative.

Another statistic which takes into account the restricted and the unrestricted estimates is also proposed:

$$LR_T^\pm = \text{sgn}[h(\hat{\varphi}) - h(\tilde{\varphi})] \sqrt{LR_T}, \quad (4)$$

where $\text{sgn}(\cdot)$ is the sign of $[h(\hat{\varphi}) - h(\tilde{\varphi})]$. This statistic can be used for constructing two-sided, equal-tailed confidence intervals and median unbiased estimates.

Finally, the $100\eta\%$ confidence interval for the half-life, which is based on impulse response analysis, is: $C_\eta(l) = \{l \in L : LR_T \leq q_\eta(c)\}$, where $q_\eta(c)$ is the η^{th} quantile of the asymptotic distribution, l is the lead time of the impulse response function and $\tilde{\varphi} = \arg \max l_T(\varphi)$ subject to $\theta_{l-0.5} = 0$. The confidence interval for the half-life can be constructed using either LR_T^\pm or LR_T .

3 Data and Preliminary Analysis

The data utilized in this analysis is extracted from the www.globalfindata.com database and includes monthly long-term bond yields and consumer price index (CPI) series for the US, UK, France and Japan spanning the period from 1876 to 2003. The exact sample period for each country is 1876:01 to 2000:06 for the US, 1934:01 to 2003:07 for the UK, 1916:01 to 2003:07 for France, and 1923:01 to 2001:08 for Japan. The end date was in each case dictated by data availability.

The long-term bond yields are preferred to short-term rates because they are closely linked to the cost of long-lived capital. Also, firms do not usually make their investment decisions on the basis of short-term rates. Indeed, to the extent that firms borrow in bond markets, long-term yields will be more informative (Fujii and Chinn, 2000). The inflation rate is defined as the annualised growth rate of the CPI which was seasonally adjusted by taking the average for the previous 12 months. Also, some data points were missing for Japan's yield during the 1940s and this was corrected using linear interpolation.

We first test for a unit root in the real interest rates using the efficient generalized least squares (GLS) version of the Dickey-Fuller (DF) test due to Elliott, Rothenberg, and Stock (1996). The results are reported in Table 1. While most unit root tests are only concerned with testing the null

that the largest root is unity against the alternative that it is less than one, the DF-GLS test tests the null against a specific alternative $H_1: \alpha < 1$ where $\alpha = 1 + c/T$. Further, using a sequence of tests of the null of a unit root against a set of stationary persistent alternatives, Elliott, Rothenberg, and Stock (1996) showed substantial power gain from the DF-GLS method over the conventional ADF test (which has low power against close alternatives so that the unit root null can seldom be rejected for highly persistent variables). The lag length is chosen using the modified AIC (MAIC) of Ng and Perron (2001) which produces the best combination of size and power. According to Table 1, one can see that the DF-GLS test rejects the unit root null hypothesis for all series at the 1% level of significance.

Table 1: Unit root tests, MUE and confidence intervals

	Rate	DF-GLS	α_{OLS}	α_{MUE}	90 _{lower}	90 _{upper}	95 _{lower}	95 _{upper}
Japan	Ex-post	[0.0005]	0.9772	0.9830	0.9763	0.9896	0.9745	0.9913
	Ex-ante	[0.0000]	0.9682	0.9754	0.9657	0.9850	0.9633	0.9875
France	Ex-post	[0.0000]	0.9760	0.9797	0.9697	0.9895	0.9675	0.9915
	Ex-ante	[0.0010]	0.9756	0.9796	0.9682	0.9908	0.9658	0.9932
US	Ex-post	[0.0020]	0.9771	0.9867	0.9798	0.9937	0.9786	0.9952
	Ex-ante	[0.0025]	0.9824	0.9924	0.9884	0.9969	0.9876	0.9978
UK	Ex-post	[0.0000]	0.9482	0.9491	0.9345	0.9635	0.9316	0.9667
	Ex-ante	[0.0005]	0.9764	0.9775	0.9665	0.9884	0.9646	0.9903

Notes: The median unbiased estimates and confidence intervals for the largest root are constructed with the grid bootstrap of Hansen (1999) using the efficiently demeaned DF-GLS statistic. The optimal lag lengths for the unit root test statistics are set according to the modified AIC of Ng and Perron (2001). p-values in square brackets.

3.1 Confidence Intervals for the Largest Root and the Half-life

Table 1 reports the median unbiased estimates of α , α_{MUE} , and the 90% and 95% MUE confidence intervals for this measure of persistence. The intervals are constructed by inverting the acceptance region of the powerful DF-GLS test of Elliott, Rothenberg, and Stock (1996). Whilst the methodology in Section 2.1 is based on an ADF regression, the extension of this method to the DF-GLS test is straightforward. Instead of working with the data in levels as in equation (2), we simply work with the GLS demeaned data in the DF-GLS regression. Moreover, the finite-sample distribution of the DF-GLS test is obtained using the grid bootstrap of Hansen (1999).

The OLS estimate of the largest root, α_{OLS} , is also reported. This estimate is based on ADF regressions and, thus, does not optimally exploit the sample information in terms of power whereas α_{MUE} , based on the DF-GLS test, does. Besides, α_{OLS} is normally treated cautiously as it is biased downwards in small samples. However, given the large size of our samples, the bias in the OLS estimate disappears almost completely.

The median unbiased (MU) estimates of the largest root are indicative of high persistence in real interest rates. The MU estimates of α range from 0.9491 to 0.9924. This latter figure corresponds to the US ex-ante real interest rate. However, none of the confidence intervals for the largest root are found to contain unity as an upper bound; although, the upper limits are in most cases near the unit root boundary. This is consistent with the results of the DF-GLS test which rejected the unit root null.

It is interesting to note that the lower bounds are close to the point estimates and are never below 0.9345. The lower bound represents the lowest degree of persistence in the real interest rate that is consistent with the data. Expressed equivalently, it represents an upper bound on how quickly deviations are corrected. If the lower bound of the confidence interval for α is large, we can conclude that the variables under scrutiny are slow to mean-revert.

On the whole, our findings are supportive of the idea that real interest rates are mean-reverting, albeit quite persistent and display near-unit-root behavior, precisely the type of behavior that will be difficult for standard tests to detect in short samples. Finally, the MUE confidence intervals from the powerful DF-GLS test appear to be rather tight and this demonstrates the potential for sharper inference from this test (Gospodinov, 2004).

The MU point estimates and confidence intervals of the half-life based on impulse response analysis are shown in Table 2. The point estimates of the half-life generally fluctuate between a low of 1.4048 and a high of almost 2.9 years. Apart from Japan, most estimates are less than 2 years, nonetheless.

Table 2: MU confidence intervals for the half-lives

	Rate	Half-life	90 _{lower}	90 _{upper}	95 _{lower}	95 _{upper}
Japan	Ex-post	2.8816	1.3243	5.2231	1.2984	6.7346
	Ex-ante	2.1204	0.9824	3.5599	0.9679	4.3080
France	Ex-post	1.8980	1.2004	4.8749	1.1438	6.0919
	Ex-ante	1.4048	1.0781	5.0297	1.0579	6.8460
USA	Ex-post	1.8399	1.5720	4.0339	1.5290	4.4561
	Ex-ante	1.8089	1.5664	6.0078	1.5058	6.7020
UK	Ex-post	1.5827	1.3032	1.9351	1.2724	2.0584
	Ex-ante	1.5616	1.2687	5.7577	1.2274	7.5519

Notes: The half-lives estimated from the impulse response functions are measured in years.

Although the point estimates are useful, what is important is the upper bound of the confidence interval. These bounds suggest a high degree of persistence, with the ex-ante real interest rate for the UK being the rate with the highest (95%) upper limit; approximately 7.56 years. The lowest upper limit is found for the UK ex-post rate. The fact that the upper bound for the UK ex-post rate is so much lower than the corresponding ex-ante rate is somewhat surprising, however. Again, the

finite half-life upper bounds indicate that the data are not consistent with a unit root in the real interest rate. In addition, while the upper bounds imply that it takes a few years for deviations to subside permanently below 0.5 in response to a shock, the lower bounds do not necessarily confirm the Fisher effect as they are always greater than one year, except for Japan which has a lower bound half-life of almost one year.

Finally, it appears that the half-life estimates do not critically depend on how real interest rates are measured, whether ex-post or ex-ante, since both are rather persistent. Under rational expectations, errors in agents' forecasts of inflation rates will tend toward zero. In addition, over long periods the errors between expected and actual inflation values are likely to be very much smaller than over shorter periods. Arguably, this may account for the fact that the difference in the persistence estimates of ex-ante and ex-post rates appears to be small.

As a final exercise, we construct the 90% and 95% confidence intervals of the impulse responses functions derived from the inversion of the LR statistic. The graphs of the first 120 responses are displayed by the solid lines in Figures 1-4. Note that the top (bottom) panels plot the monthly responses for the ex-post (ex-ante) rates. In addition, the dashed and dotted lines in the panels on the left (right) plot the 90% (95%) confidence intervals. While impulse response analysis can be performed for even longer horizons, we report results up to 10 years since this is quite a close approximation to the infinite horizon. In all cases, real interest rates have zero long-run persistence, confirming the existence of long-run mean-reversion and, most importantly, the absence of a unit root. The upper limits of the confidence intervals of the impulse response functions suggest that one quarter of the adjustment is completed within 3 years.

The process of convergence deserves further attention, however. This process appears to exhibit some nonlinearity in its adjustment due to short-term overreacting. Specifically, the impulse responses are all hump-shaped, with initial shock intensification before eventual dissipation. Given that the shock impact magnifies in the initial few months rather than diminishes, the maximum response cannot be felt until a few years as a consequence.

Figure 1: US

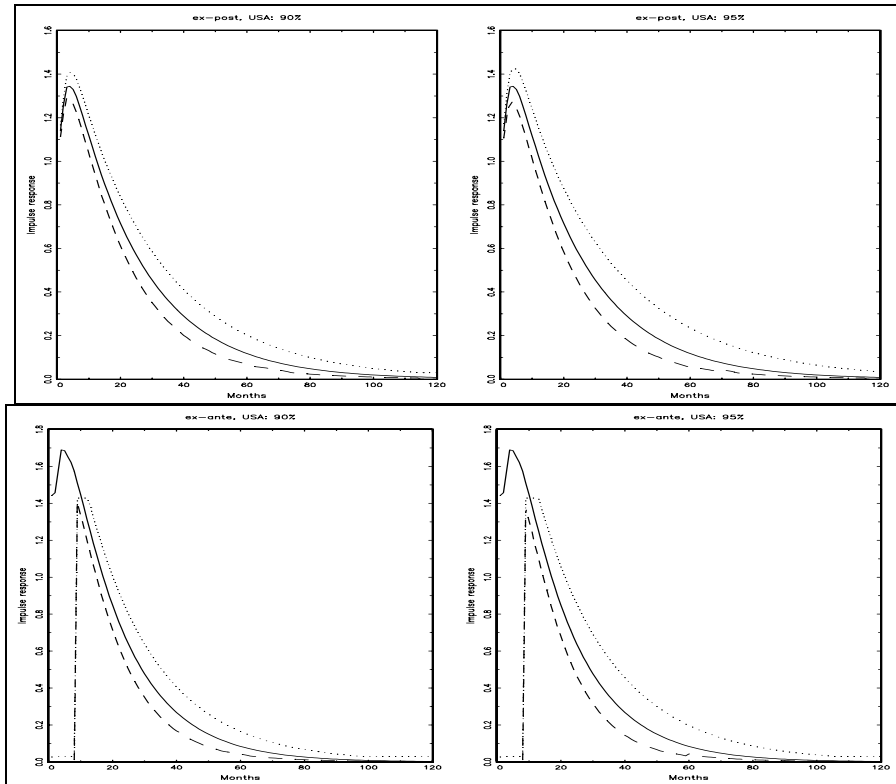


Figure 2: UK

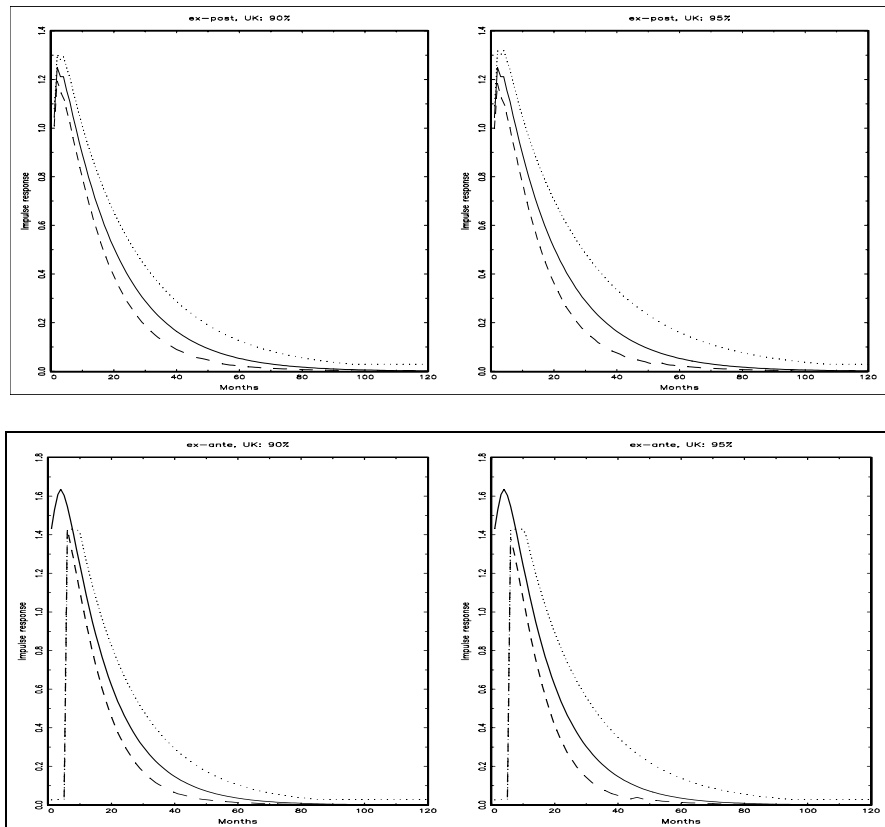


Figure 3: France

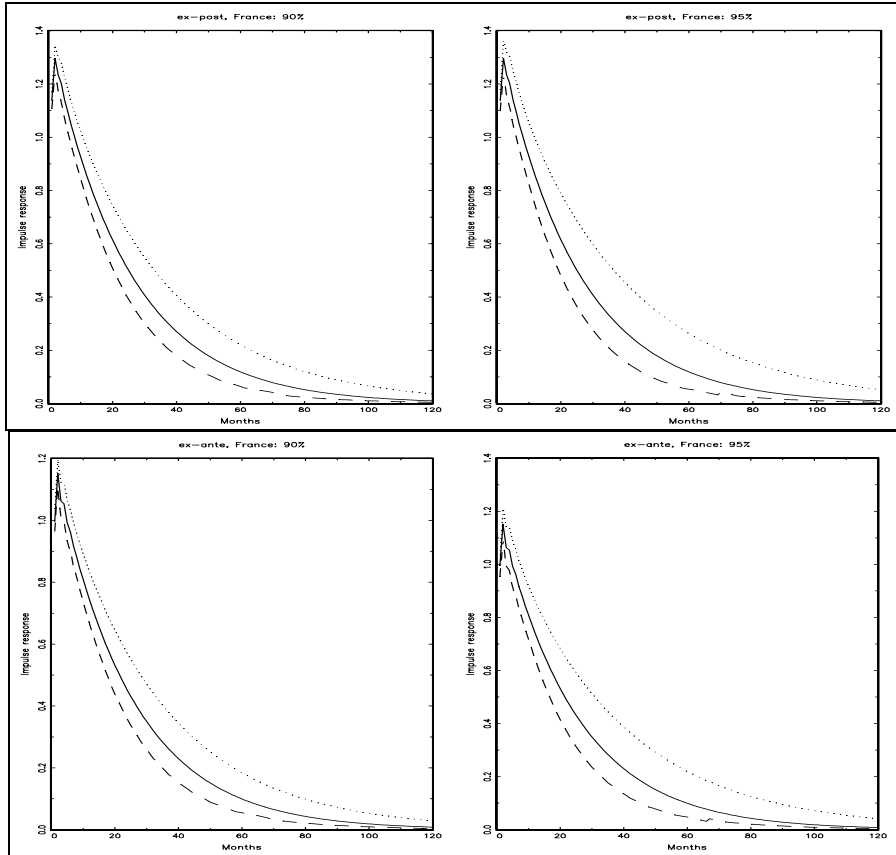
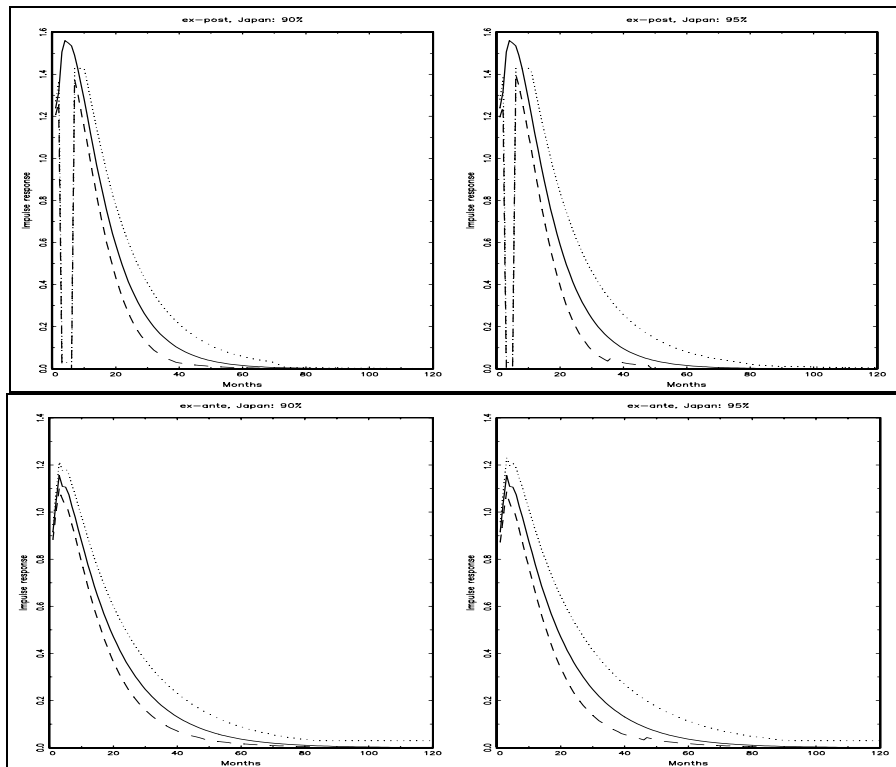


Figure 4: Japan



4 Conclusion

The Fisher effect embodies the hypothesis that inflation and the nominal interest rate move one-for-one in the long-run so that the real interest rate is stationary. Starting with Fisher and extending to the present, this concept has found limited empirical support. Typically, the literature cannot reject the hypothesis that the real interest rate is a realization of a unit root process. However, unit root tests may have low power to reject the unit root null hypothesis due to the relatively small number of observations that are normally used.

Recognising the importance of the real interest rate for the Fisher hypothesis, monetary policy and theoretical modelling, we have examined the time-series properties of four major monthly real interest rates using samples that range from 70 to 125 years. The expectation is that tests based on long data series will have more power to reject a unit root than those using short samples. The use of long spans of data is also motivated by studies that find better support for long-run economic relationships using such data.

We have also investigated the persistence of real interest rates through the computation of median unbiased point estimates and confidence intervals for the half-lives of deviations for DF-GLS regressions. The results indicate the absence of a unit root in ex-ante and ex-post real interest rates. In fact, the unit root null hypothesis is rejected at the 1% level of significance. Nonetheless, the confidence intervals for the half-lives suggest that these rates are rather persistent since it may take up to 7.55 years (in the case of the UK) for half of the shock to die out. Thus, even though we reject the unit root hypothesis, the intervals for the largest root and the half-life indicate that real interest rates are highly persistent.

The persistence of real interest rates is problematic not just for the Fisher hypothesis and the determination of the effects of monetary and fiscal policies but also for a number of models (e.g. the CCAPM, the neoclassical growth model with explicitly optimizing agents and various models of the monetary transmission mechanism).

The CCAPM, in particular, implies that the growth rate of consumption and the real interest rate should have similar time-series characteristics (Rose, 1988). Still, the growth rate of consumption has been found to contain no unit root and does not exhibit the persistence evident in real interest rates (Rapach and Wohar, 2004). The differences in the degree of persistence means that the Euler equation derived from the CCAPM cannot be expected to hold in the presence of a persistent real interest rate. Finally, recent research by Garcia and Perron (1996), Bai and Perron (2003), and Lai (2004) suggest that real interest rates are not unit root once a mean shift is allowed for. However, structural breaks are as problematic as persistence since consumption growth does not display evidence of structural breaks like the real interest rate (Rapach and Weber, 2004).

References

- [1] Andrews, D. W. K., and H. Y. Chen (1994). Approximately Median-unbiased Estimation of Autoregressive Models. *Journal of Business Economic and Statistics*, 12, 187-204.
- [2] Bai, J., and P. Perron (2003). Computation and Analysis of Multiple Structural Change Models. *Journal of Applied Econometrics*, 18, 1-22.
- [3] Cheung, Y. W., and K. S. Lai (2000). On the Purchasing Power Parity Puzzle. *Journal of International Economics*, 52, 321-330.
- [4] Crowder, W. J., and D. L. Hoffman (1996). The Long-run Relationship between Nominal Interest Rates and Inflation: the Fisher Equation Revisited. *Journal of Money, Credit and Banking*, 28, 102-118.
- [5] Dickey, D. A., and W. A. Fuller (1979). Distribution of the Estimators for Autoregressive Time Series with a Unit Root. *Journal of the American Statistical Association*, 74, 427-431.
- [6] Elliott, G., Rothenberg, T., and J. H. Stock (1996). Efficient Tests for an Autoregressive Unit Root. *Econometrica*, 64, 813-836.
- [7] Evans, M. D., and K. K. Lewis (1995). Do Expected Shifts in Inflation Affect Estimates of the Long-run Fisher Relation? *Journal of Finance*, 50, 225-253.
- [8] Fisher, I. (1930). *The Theory of Interest*, Macmillan, New York.
- [9] Frankel, J. A. (1986). International Capital Mobility and Crowding-out in the U.S. Economy: Imperfect Integration of Financial Markets or Goods Markets?, in *How Open is the U.S. Economy?*, R. W. Hafer (Ed.), Lexington Books, Lexington, Massachusetts.
- [10] Frankel, J. A. (1990). Zen and the Art of Modern Macroeconomics: the Search for Perfect Nothingness, in *Monetary Policy for a Volatile Global Economy*, W. Haraf and T. Willet (Ed.), American Enterprise Institute, Washington D.C.
- [11] Fujii, E., and M. D. Chinn (2000). *Fin de Siècle Real Interest Parity*. NBER WP 7880.
- [12] Garcia, R., and P. Perron (1996). An Analysis of the Real Interest Rate under Regime Shifts. *Review of Economics and Statistics*, 78, 111-125.
- [13] Goodwin, B. K., and T. J. Grennes (1994). Real Interest Rate Equalization and the Integration of International Financial Markets. *Journal of International Money and Finance*, 13, 107-124.
- [14] Gospodinov, N. (2004). Asymptotic Confidence Intervals for Impulse Responses of Near-integrated processes: An Application to Purchasing Power Parity. *Econometrics Journal*, 7, 505-527.
- [15] Gottschalk, J. (2001). Measuring Expected Inflation and the Ex-ante Real Interest Rate in the Euro Area using Structural VARs. Kiel Institute for World Economics WP 1067.
- [16] Hansen, B. (1999). The Grid Bootstrap and the Autoregressive Model. *Review of Economics and Statistics*, 81, 594-607.
- [17] Karanasos, M., Sekioua, S. H., and N. Zeng (2006). On the Order of Integration of Monthly US Ex-ante and Ex-post Real Interest Rates: New Evidence from over a Century of Data. *Economics Letters*, 90, 163-169.
- [18] Koustas, Z., and A. Serletis (1999). On the Fisher Effect. *Journal of Monetary Economics*, 44, 105-130.
- [19] Lai, K. S. (1997). Long-term Persistence in the Real Interest Rate: Some Evidence of a Fractional Unit Root? *International Journal of Finance and Economics*, 2, 225-235.
- [20] Lai, K. S. (2004). On Structural Shifts and Stationarity of the Ex-ante Real Interest Rate. *International Review of Economics and Finance*, 13, 217-228.
- [21] Lothian, J. R., and M. P. Taylor (1996). Real Exchange Rate Behavior: The Recent Float from the Perspective of the Last two Centuries. *Journal of Political Economy*, 104, 488-510.

- [22] Lothian, J. R., and M. P. Taylor (2000). Purchasing Power Parity over two Centuries: Strengthening the Case for Real Exchange Rate Stability: A Reply to Cuddington and Liang. *Journal of International Money and Finance*, 19, 759-764.
- [23] Lucas, R. E. (1978). Asset Prices in an Exchange Economy. *Econometrica*, 46, 1429-1445.
- [24] MacDonald, R., and P. Murphy (1989). Testing for the Long-run Relationship between Nominal Interest Rates and Inflation Using Cointegration Techniques. *Applied Economics*, 21, 439-447.
- [25] Mishkin, F. S. (1992). Is the Fisher Effect for Real? A Re-examination of the Relationship between Inflation and Interest Rates. *Journal of Monetary Economics*, 30, 195-215.
- [26] Ng, S., and P. Perron (2001). Lag Length Selection and the Construction of Unit Root Tests with Good Size and Power. *Econometrica*, 69, 1519-1554.
- [27] Perron, P. (1989). Testing for a Random Walk: A Simulation Experiment of Power when the Sampling Interval is Varied, in *Advances in Econometrics and Modelling*, 47-68, B. Raj (Ed.), Kluwer Academic Publishers, Dordrecht.
- [28] Phylaktis, K. (1999). Capital Market Integration in the Pacific Basin Region: An Impulse Response Analysis. *Journal of International Money and Finance*, 19, 267-287.
- [29] Pierse, R. G., and A. J. Snell (1995). Temporal Aggregation and the Power of Tests for a Unit Root. *Journal of Econometrics*, 65, 333-345.
- [30] Rapach, D. E., and C. E. Weber (2004). Are Real Interest Rates Really Nonstationary? New Evidence from Tests with Good Size and Power. *Journal of Macroeconomics*, 26, 409-430.
- [31] Rapach, D. E., and M. E. Wohar (2002). Testing the Monetary Model of Exchange Rate Determination: New Evidence from a Century of Data. *Journal of International Economics*, 58, 359-385.
- [32] Rapach, D. E., and M. E. Wohar (2004). The Persistence in International Real Interest Rates. *International Journal of Finance and Economics*, 9, 339-346.
- [33] Romano, J. P., and M. Wolf (2001). Subsampling Intervals in Autoregressive Models with Linear Time Trends. *Econometrica*, 69, 1283-1314.
- [34] Rose, A. K. (1988). Is the Real Interest Rate Stable? *Journal of Finance*, 43, 1095-1112.
- [35] Sarno, L., and M. P. Taylor (2003). *Economics of Exchange Rates*, Cambridge University Press, Cambridge.
- [36] Shiller, R. J., and P. Perron (1985). Testing the Random Walk Hypothesis: Power Versus Frequency of Observation. *Economics Letters*, 18, 381-386.
- [37] St-Amant, P. (1996). Decomposing U.S. Nominal Interest Rates into Expected Inflation and Ex-ante Real Interest Rates using Structural VAR Methodology. Bank of Canada WP 96-2.
- [38] Taylor, A. M. (2002). A Century of Purchasing Power Parity. *Review of Economics and Statistics*, 84, 139-150.
- [39] Tsay, W. J. (2000). Long Memory Story of the Real Interest Rate. *Economics Letters*, 67, 325-330.
- [40] Valente, G. (2003). Monetary Policy Rules and Regime Shifts. *Applied Financial Economics*, 13, 525-535.
- [41] Wallace, M. S., and J. T. Warner (1993). The Fisher Effect and the Term Structure of Interest Rates: Tests of Cointegration. *Review of Economics and Statistics*, 75, 320-324.