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# Finite-Sample Performance of Structural Change Tests in Cointegrated Relationships: A Monte Carlo Analysis

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## **Abstract**

We present Monte Carlo simulation experiments to investigate the finite sample properties of structural change tests in cointegrated relationships. These tests are computed using three alternative efficient estimators of cointegrating vectors. It is shown that the tests have overall accurate size and good power performance with a slight out-performance of the fully modified and the leads and lags estimators based tests. However, our simulation results reveal substantial power decrease for all the tests due to higher error serial correlation.

*JEL Classifications:* C12, C14, C15.

*Keywords:* Structural change, cointegration, central limit theory, nonparametric estimation, Monte Carlo simulation.

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# 1 Introduction

There has been extensive work on statistical procedures designed to test for structural change in regression coefficients with stationary series. One important class of tests on structural change are the tests based on the generalized fluctuation test type, which includes the CUSUM and the MOSOM tests and the fluctuation test (Kuan and Hornik, 1995). An alternative class of tests, which includes the Chow and the sup-F tests, was developed by Hansen (1992) and Andrews and Ploberger (1994).

It is fundamental, in particular in terms of economic applications, to extend the analysis to more general dynamic models that include integrated processes and cointegrated systems. Hansen (1992, 1997) developed tests for structural breaks in cointegrated relations with unknown break points. He derived the asymptotic distributions and the critical values of these tests based on the fully modified estimator in general models that include stochastic and deterministic trends.

Gregory, Nason and Watt (1996) investigated the finite-sample performance of these tests in the context of a linear-quadratic model with fully modified (FM) estimation. Although it was found that FM estimators have good finite-sample properties in terms of bias and coverage probabilities (see Phillips and Loteran, 1991), the results in Stock and Watson (1993) suggest some shortcomings of these estimators.

Thus, the purpose of this paper is to present a performance assessment of the structural change tests in cointegration, using not only the FM estimator but also alternative efficient estimators. We conduct Monte Carlo experiments to evaluate their size and power performance in finite samples.

Consider the following system:

$$y_t = \alpha + \beta X_t + u_{1t}, \quad (1)$$

$$\Delta X_t = u_{2t}, \quad (2)$$

where  $\Delta$  is the difference operator and both series  $\{y_t\}$  and  $\{X_t\}$  are integrated I(1).

We assume that the partial sum process constructed from the innovation sequence  $u_t = (u_{1t}, u_{2t})'$  satisfies the invariance principle (Billingsley, 1968):

$$\frac{1}{\sqrt{T}} \sum_{t=1}^{[Tr]} u_t \Rightarrow B(r) = \begin{pmatrix} B_1(r) \\ B_2(r) \end{pmatrix},$$

where " $\Rightarrow$ " signifies weak convergence and  $B(r)$  is the Brownian motion with covariance matrix  $\Omega$  defined below.

The ordinary least squares (OLS) estimator of  $\beta$  is:

$$\hat{\beta}_{OLS} = \frac{\sum_{t=1}^T (y_t X_t)}{\sum_{t=1}^T X_t^2} = \beta + \frac{\sum_{t=1}^T (y_t u_{1t})}{\sum_{t=1}^T X_t^2}.$$

Using central limit theory (see Phillips and Durlauf, 1986) we obtain:

$$\frac{1}{T^2} \sum_{t=1}^T X_t^2 \Rightarrow \int_0^1 B_2^2,$$

$$\frac{1}{T} \sum_{t=1}^T X_t u_{1t} \Rightarrow \int_0^1 B_2 dB_1 + \gamma_{1,2},$$

where  $\gamma_{1,2}$  is defined below.

The above result shows that  $\hat{\beta}_{ols}$  is super-consistent (it converges at rate T) but its limiting distribution is non-Gaussian and is biased. This may be explained by the long-run correlation and the cross correlation between the regressor and the regression error, which creates an obstacle to inference for the cointegration restrictions since conventional Chi-squared type tests are no longer valid. Thus, in general, OLS estimation of cointegration is inefficient.

In this paper, our objective is to assess the performance of three different structural change tests, namely, Andrews-Quandt F<sub>sup</sub>, Andrews-Ploberger F<sub>mean</sub>, and Nyblom-Hansen L<sub>C</sub>, which are calculated in cointegration models based on three different estimators.

These are the FM estimation, the leads and lags estimation, and the canonical cointegrating regression. We use Monte Carlo experiments to evaluate the size and the power of each test. We begin with a brief description of these methods. For more detail on the properties of the estimators we refer the reader to the original papers of Phillips and Hansen (1990), Saikkonen (1991), Park (1992) and Stock and Watson (1993).

## 2 Cointegration and Structural Change

A number of efficient estimation methods of cointegrated vectors have been developed in the literature. Phillips and Hansen introduced the fully modified estimator (FME). Their method is semi-parametric and requires a two-step estimation procedure. An alternative method, which is nonparametric, was studied by Saikkonen (1991), who defined asymptotic efficiency based on the concentration of the limiting distribution of the estimator. Based on this definition he proposed a class

of asymptotically efficient estimators. His method consists of adding leads and lags of the regressor in the cointegration regression.

A similar idea can also be found in Stock and Watson (1993), who extended the analysis to higher order integrated variables. Park (1992) suggested a transformation of the regressor  $X_t$  and the regressand  $y_t$ , which eliminates the endogeneity caused by the long-run correlation between the regressor and the regression error. The transformed model is called canonical cointegration regression and the resulting estimators are asymptotically efficient. The unifying property of these estimators is that they have a mixture normal asymptotic distribution which fulfils the requirement of the test procedure:

$$(\hat{\theta} - \theta) [diag(\sqrt{T}, T)] \Rightarrow \left( B_{1,2}, \left( \int_0^1 dB_{1,2} B_2 \right) \right) \left( \int_0^1 \tilde{B}_2 \tilde{B}_2' \right)^{-1}, \quad (3)$$

where  $\theta = (\alpha, \beta)'$ ,  $\tilde{B}_2$  is the limiting process of the partial sum constructed from  $\tilde{X}_t = (1, X_t)'$  and  $B_{1,2} = B_1 - \omega_{12} \omega_{22}^{-1} B_2$  (the  $\omega$ 's are defined below).

To implement these procedures, we need to estimate the covariance matrix. Let  $\Omega$  denote the covariance matrix of the limiting process  $B(r)$ , then we have:

$$\Omega = 2\pi f_{uu}(0) = \lim_{T \rightarrow \infty} \frac{1}{T} \sum_{t=1}^T \sum_{\tau=1}^T E(u_t u_\tau') = \begin{bmatrix} \omega_{11} & \omega_{12} \\ \omega_{21} & \omega_{22} \end{bmatrix},$$

where  $f_{uu}(\cdot)$  is the spectral density of  $\{u_t\}$  and  $\Omega$  is partitioned in conformity with  $u_t$ . We define the following long-run covariance matrices :

$$\Sigma = \lim_{T \rightarrow \infty} \frac{1}{T} \sum_{t=1}^T E(u_t u_t') = \begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{bmatrix}, \text{ and } \Gamma = \lim_{T \rightarrow \infty} \frac{1}{T} \sum_{t=1}^T \sum_{\tau=1}^t E(u_t u_\tau') = \begin{bmatrix} \gamma_{11} & \gamma_{12} \\ \gamma_{21} & \gamma_{22} \end{bmatrix}.$$

Consider  $\omega_{1,2} = \omega_{11} - \omega_{12} \omega_{21} \omega_{22}^{-1}$  and  $\gamma_{1,2} = \gamma_{21} - \gamma_{22} \omega_{21} \omega_{22}^{-1}$  which is the bias due to endogeneity of the variables. We use a heteroskedasticity and autocorrelation consistent (HAC) covariance matrix estimator following Andrews and Monahan (1992). This estimator is based on pre-whitened kernel estimators with an auto-regression employed in the pre-whitening stage. The structural change tests considered are computed from the estimators of cointegration vectors which, in turn, are constructed from these HAC variance and covariance estimators.

Consider the transformed dependent variable:

$$y_t^* = y_t - \omega_{12} \omega_{22}^{-1} u_{2t}.$$

The fully modified estimator of  $\theta$  and the associated score are given by

$$\hat{\theta}_{FME} = \left( \sum_{t=1}^T (y_t^* \tilde{X}_t' - (0, \gamma_{1.2})') \right) \left( \sum_{t=1}^T \tilde{X}_t \tilde{X}_t' \right)^{-1}.$$

The FM residuals are  $\hat{u}_{1t}^* = y_t^* - \hat{\theta}_{FME} \tilde{X}_t$ ,  $\hat{u}_{2t} = \Delta X_t$ , and we have  $\frac{1}{T} \sum_{t=1}^T \tilde{X}_t \hat{u}_{1t} = \begin{pmatrix} 0 \\ \hat{\gamma}_{1.2} \end{pmatrix}$ .

Next, we present the structural change tests investigated in this paper. The literature on structural change is extensive and fast growing. In recent articles, Zeileis (2005) and Zeileis, Leisch, Kleiber and Hornik (2005) developed three classes of structural change tests which have been receiving much attention from both applied statisticians and econometricians. The authors studied these tests and showed that they can be embedded in classes which represent special cases of generalized M-fluctuation tests. These tests are based on the same central limit theory but differ in the functionals for capturing excessive fluctuations. Our intention, however, is to assess the performance of the stability tests in a cointegrated system of time series variables based on alternative estimation methods.

Define the score  $\hat{s}_t = \left( \tilde{X}_t \hat{u}_{1t}^* - \begin{pmatrix} 0 \\ \hat{\gamma}_{1.2} \end{pmatrix} \right)$ , and the cumulative score  $S(\lambda) = \sum_{t=1}^{[T\lambda]} \hat{s}_t$ , where  $\lambda$  is in some

compact subset of  $(0, 1)$ .

The parameter  $\theta$ , which may be time dependent, is constant under the null hypothesis  $H_0: \theta_1 = \theta_2$ . Our treatment of the alternative hypothesis differs for each test. First, we consider the alternative  $H_1: \theta_t = \begin{cases} \theta_1 & \text{if } t \leq [T\lambda] \\ \theta_2 & \text{if } t > [T\lambda] \end{cases}$ , for some  $\lambda$  between 0 and 1.

We proceed by defining the following test statistic:

$$F(\lambda) = S(\lambda)' V(\lambda) S(\lambda) \hat{\omega}_{1.2},$$

where  $V(\lambda) = M(\lambda) - M(\lambda)M(1)^{-1}M(\lambda)$ , and  $M(\lambda) = \sum_{t=1}^{[T\lambda]} \tilde{X}_t \tilde{X}_t'$ .

Following standard central limit theory, it is shown that  $F(\lambda)$  has a Chi-squared asymptotic distribution when the break point  $\lambda$  is known. This result can be extended to functional central limit theory when  $\lambda$  is unknown. In this case, we calculate a sequence of  $F(\lambda)$  statistics over all the values of  $\lambda$  and the test statistic is given by the largest value.

This test is quite popular in the literature of structural breaks with integrated variables. The idea was initially developed by Quandt (1960) and then extended by Hansen (1992) and Andrews (1993). Gregory, Nason and Watt (1996) applied the test in the context of linear-quadratic models to investigate its finite sample performance.

The limitation of the test is the need to specify a compact set that does not include the endpoints in order to avoid the divergence of the test statistic as shown by Andrews (1993). In this paper, we select the set  $\Lambda = [.15, .85]$ . This is the standard selection in most empirical studies.

In the second test, we assume that  $\theta_t$  is given by a martingale difference sequence,  $\theta_t = \theta_{t-1} + \eta_t$ , where the conditional mean of  $\eta_t$  is zero and its variance is given by  $m^2 G_t$ , and  $G_t$  is a known covariance array that measures the parameter stability at time t.

Under the null hypothesis  $H_0$  we have:  $\text{Var}(\eta_t) = 0$ . Under the alternative hypothesis  $H_2$  we have:  $\text{Var}(\eta_t) \neq 0$ .

This implies that under  $H_2$  we have:  $m^2 > 0$ ,  $G(T\lambda) = (\hat{\omega}_{1,2} \otimes V(T\lambda))^{-1}$  and the test statistic is given by  $F_{\text{mean}} = \frac{1}{T} \sum_{t=1}^T F(\lambda_t)$ . Although this test does not require trimming, we restrict the values of  $\lambda$  between 0.15 and 0.85.

The third test is based on the same null hypothesis as previously, and the alternative hypothesis is  $H_3: m^2 > 0$ ,  $G(T\lambda) = (\hat{\omega}_{1,2} \otimes M(1))^{-1}$ . The test statistic is given by  $L_c = \frac{1}{T} \text{tr} \left( M(1)^{-1} \sum_{t=1}^T S_t \hat{\omega}_{1,2}^{-1} S_t' \right)$ . This is similar to the Lagrange-Multiplier test, it is easy to compute and does not require a compact subset for  $\lambda$ . However, it should be noted, as described in Zeileis (2005), that the functional used by the  $L_c$  test is designed for multiple break alternatives.

The distribution theory and critical values for these tests can be found in Hansen (1997). We compute these alternative tests using estimators from the canonical cointegration regression and the leads and lags estimation, which are efficient methods. The distribution of the tests is given in equation (3) and is similar to FME.

### 3 Monte Carlo Simulation Results

In this section, we present Monte Carlo simulation experiments in order to assess the finite sample properties of structural break tests computed from alternative estimators of the cointegration relationships. The model is specified by the following equations:

$$y_t = \alpha + \beta_t X_t + u_{1t}, \tag{4}$$

$$X_t = X_{t-1} + u_{2t}, \tag{5}$$

where  $u_t = \begin{pmatrix} u_{1t} \\ u_{2t} \end{pmatrix} = \rho u_{t-1} + e_t$ , and  $e_t = \begin{pmatrix} e_{1t} \\ e_{2t} \end{pmatrix} \rightarrow N \left[ \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma^2 & \sigma_{12} \\ \sigma_{12} & \sigma^2 \end{pmatrix} \right]$ .

Our simulation design is similar to Gregory (1994). In all the regressions, we include a constant in the data generating process. We consider 5000 replications in each simulation experiment with three alternative sample sizes: T = 100, 200, and 500, starting with  $u_{10} = u_{20} = 0$ . We use Gauss programs to generate the pseudo-normal innovations. We compute the tests for structural change based on FME, canonical cointegration regression (CCR) and leads and lags estimation (LLE). These tests were presented by Hansen (1992) using only fully modified estimation.

Thus, we obtain nine tests: FME-F<sub>sup</sub>, FME-F<sub>mean</sub>, FME-L<sub>C</sub>, CCR-F<sub>sup</sub>, CCR-F<sub>mean</sub>, CCR-L<sub>C</sub>, LLE-F<sub>sup</sub>, LLE-F<sub>mean</sub>, and LLE-L<sub>C</sub>. For each experiment, we use the five percent asymptotic critical values to record the rejection frequencies of each test. We follow Gregory, Nason and Watt (1996) in our choice of the lag length of the estimators. We start with lag length equals 8 and we test downward until the last lag is significant at the 5% level using normal values. We apply the same procedure to find the lead length.

Next, we need to estimate the variance and covariance parameters as follows. We estimate model equation (1) by OLS to get  $\hat{\theta}_{OLS}$  and the corresponding residuals  $\hat{u}_{1t} = y_t - \hat{\theta}_{OLS} \tilde{X}_t$ . The second element of the residual vector is computed from  $\hat{u}_{2t} = \Delta X_t$ .

The parameters  $\omega_{1,2}$  and  $\gamma_{1,2}$  can now be estimated from the residuals  $\hat{u}_t$  based on a pre-whitened kernel estimator following the Andrews plug-in bandwidth method as suggested in Andrews (1991) and in Andrews and Monahan (1992). In their study these authors found the pre-whitening to be very effective in reducing bias, improving confidence interval coverage probabilities, and in rescuing over-rejection of t-statistics constructed using kernel-HAC estimators. Thus, the bandwidth parameters in our simulation experiments are a function of the data.

Following Hansen, we fit a first order auto-regression to the residuals, in order to provide an initial approximation to the spectral density at frequency zero, which is then used to estimate the optimal bandwidth parameter. We consider the model  $\hat{u}_t = \hat{\rho} \hat{u}_{t-1} + \hat{e}_t$ , where  $\hat{e}_t$  is a vector of white noise residuals. We compute the kernel estimators as follows:

$$\hat{\omega}_{1,2} = \sum_{|i| < T} k \left( \frac{i}{M} \right) \frac{1}{T} \sum_{t=i+1}^T \hat{e}_{t-i} \hat{e}_t'$$

$$\hat{\gamma}_{1,2} = \sum_{i=0}^T k \left( \frac{i}{M} \right) \frac{1}{T} \sum_{t=i+1}^T \hat{e}_{t-i} \hat{e}_t'$$

where  $k(\cdot)$  is the quadratic spectral kernel, given by

$$k\left(\frac{i}{M}\right) = \frac{25}{12\pi^2} \left(\frac{i}{M}\right)^{-2} \left\{ \frac{\sin \frac{6\pi i}{5M}}{\frac{6\pi i}{5M}} - \cos \frac{6\pi i}{5M} \right\}.$$

The long-run covariance matrix that we use in the experiments is obtained from a pre-filtered quadratic spectral kernel with an auto-regression of order one used in the pre-filtering. The idea is to remove some of the dependence in the errors, and then we compute  $\hat{\omega}_{1,2}$  and  $\hat{\gamma}_{1,2}$  from these whitened residuals. In the simulation hyper-parameter settings, we chose a weighting matrix in order to derive the bandwidth parameter growth rate that minimizes the mean square error of the spectral estimator.

The optimal bandwidth parameter  $M$  depends on this weighting matrix and on the smoothness properties of the kernel. Let  $c(QS)$  be the normalized curvature of the quadratic spectral kernel. We estimate this term in our simulation experiments as follows:

$$\hat{c}(QS) = \frac{\sum w_i \frac{4\hat{\rho}_i^2 \hat{\sigma}_i^4}{(1-\hat{\rho})^8}}{\sum w_i \frac{\hat{\sigma}_i^4}{(1-\hat{\rho})^4}},$$

where  $(\hat{\rho}_i, \hat{\sigma}_i^2)$  are the parameter estimates from the AR(1) representation of each element in  $\hat{e}_i$ . We follow Andrews and Monahan (1992) in setting the weights corresponding to the slope coefficients equal to unity, and the weight corresponding to the intercept equal to zero. Now the optimal bandwidth parameters for the quadratic spectral kernel estimator may be given by

$$M_T = 1.3321 (\hat{c}(QS) T)^{1/5}.$$

Alternative procedures for computing the bandwidth parameters can be found in Newey and West (1994), and also in Hirukawa (2006).

In Table 1, we present the size performance of the tests. The parameter settings are  $\alpha = 2$ ,  $\beta = 1$ , and the innovations  $e_{1t}$  and  $e_{2t}$  are normally distributed with mean zero and unit variance and their covariance is equal to 0.4. The regression error terms,  $u_{1t}$  and  $u_{2t}$  are modelled by a vector auto-regression of order 1, with alternative correlation coefficients settings. We set the correlation coefficient  $\rho = 0.2$ , and  $\rho = 0.8$ , respectively.

Overall the simulation results cast little size distortion. Most of the test sizes are close to their (5%) asymptotic critical value for all the range of sample sizes.

**Table 1: Size Simulation of the Stability Tests**


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		$\rho = 0.2$	$\rho = 0.8$	$\rho = 0.2$	$\rho = 0.8$	$\rho = 0.2$	$\rho = 0.8$
		<b>T=100</b>	<b>T=100</b>	<b>T=200</b>	<b>T=200</b>	<b>T=500</b>	<b>T=500</b>
<b>FME</b>	F <sub>mean</sub>	0.065	0.046	0.079	0.033	0.062	0.061
	F <sub>sup</sub>	0.053	0.029	0.064	0.053	0.063	0.044
	L <sub>c</sub>	0.033	0.033	0.051	0.037	0.042	0.057
<b>CCR</b>	F <sub>mean</sub>	0.087	0.046	0.088	0.045	0.070	0.038
	F <sub>sup</sub>	0.071	0.055	0.054	0.048	0.072	0.036
	L <sub>c</sub>	0.044	0.042	0.047	0.068	0.068	0.060
<b>LLE</b>	F <sub>mean</sub>	0.059	0.059	0.056	0.052	0.047	0.062
	F <sub>sup</sub>	0.065	0.047	0.062	0.053	0.081	0.044
	L <sub>c</sub>	0.072	0.071	0.054	0.064	0.069	0.046

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With stronger error serial correlation in the data,  $\rho=0.8$ , it can be seen that this does not distort the test sizes further; they remain fairly close to their asymptotic value. A similar result, but restricted to the fully modified estimation based tests, can be found in Gregory, Nason and Watt (1996).

We turn to the power performance of the tests. Since there is no case of significant over-rejection in our size simulations, we do not need to size-correct the power calculations. We consider the following data generating process in our experiments:

$$y_t = 1 + b X_t + u_{1t},$$

$$\Delta X_t = u_{2t},$$

where  $b = \begin{cases} 1 & \text{if } t \leq [T\lambda] \\ 2 & \text{if } t > [T\lambda] \end{cases}, t = 1, \dots, T.$

In Tables 2, 3, and 4 below we report the power simulations for three alternative break points  $d = 0.3, 0.5,$  and  $0.8.$

We set the break point at  $d = 0.3,$  and the results in Table 2 indicate a good performance of all the tests in case of weak error serial correlation. In particular, the FME- $F_{\text{sup}}$  and LLE-  $F_{\text{mean}}$  tests reject stability at 93.4% and 90.7%, respectively. With smaller sample-size data, the rejection is slightly less frequent at 77.3% and 76.1%, respectively.

**Table 2: Power Simulation of the Stability Tests ( $d = 0.3$ )**

		$\rho = 0.2$	$\rho = 0.2$	$\rho = 0.8$	$\rho = 0.8$
		<b>T = 100</b>	<b>T = 200</b>	<b>T = 100</b>	<b>T = 200</b>
<b>FME</b>	$F_{\text{mean}}$	0.724	0.895	0.270	0.381
	$F_{\text{sup}}$	0.773	0.934	0.359	0.427
	$L_c$	0.656	0.825	0.294	0.391
<b>CCR</b>	$F_{\text{mean}}$	0.617	0.732	0.186	0.374
	$F_{\text{sup}}$	0.659	0.780	0.241	0.353
	$L_c$	0.452	0.692	0.244	0.337
<b>LLE</b>	$F_{\text{mean}}$	0.761	0.907	0.356	0.405
	$F_{\text{sup}}$	0.755	0.853	0.289	0.339
	$L_c$	0.627	0.803	0.268	0.354

However, when the errors are highly correlated, the ability of the tests to reject the false null hypothesis of no structural change is no longer convincing. For instance, the power of the CCR-  $F_{\text{mean}}$  test decreases to 18.6%, and only FME- $F_{\text{sup}}$  and LLE- $F_{\text{mean}}$  tests can still reach the 40% level. It can also be noticed that, in general, the CCR-based tests are not as powerful as the competing tests.

In Table 3 we consider other power simulation experiments that involve a break point  $d = 0.5$ . The most noticeable result is again the sharp decrease in power with increased error correlation, and the poor performance of the CCR-based tests.

**Table 3: Power Simulation of the Stability Tests ( $d = 0.5$ )**

		$\rho = 0.2$	$\rho = 0.2$	$\rho = 0.8$	$\rho = 0.8$
		<b>T = 100</b>	<b>T = 200</b>	<b>T = 100</b>	<b>T = 200</b>
<b>FME</b>	$F_{\text{mean}}$	0.754	0.883	0.317	0.424
	$F_{\text{sup}}$	0.712	0.910	0.326	0.446
	$L_c$	0.732	0.877	0.258	0.373
<b>CCR</b>	$F_{\text{mean}}$	0.661	0.726	0.229	0.381
	$F_{\text{sup}}$	0.625	0.800	0.284	0.419
	$L_c$	0.574	0.747	0.272	0.398
<b>LLE</b>	$F_{\text{mean}}$	0.773	0.881	0.373	0.493
	$F_{\text{sup}}$	0.752	0.862	0.305	0.380
	$L_c$	0.687	0.785	0.238	0.315

The FME-based tests perform relatively worse than the LLE-based tests when the break point is set at the end of the data. This result is shown in Table 4, with  $d = 0.8$ .

It is also the case that all the tests lose their power to detect structural breaks when the model error terms are highly correlated. One possible explanation is the fact that these tests are unable to discriminate between a highly serial correlated error and a random walk in the intercept, which is equivalent to the former when  $\rho$  approaches one. We conducted additional simulation experiments with lower values of residual serial correlation and the results show that their power increases considerably, in particular for the FME based tests.

**Table 4: Power Simulation of the Stability Tests ( $d = 0.8$ )**

		$\rho = 0.2$	$\rho = 0.2$	$\rho = 0.8$	$\rho = 0.8$
		<b>T = 100</b>	<b>T = 200</b>	<b>T = 100</b>	<b>T = 200</b>
<b>FME</b>	F <sub>mean</sub>	0.719	0.872	0.254	0.346
	F <sub>sup</sub>	0.691	0.895	0.365	0.426
	L <sub>c</sub>	0.702	0.797	0.250	0.351
<b>CCR</b>	F <sub>mean</sub>	0.643	0.748	0.237	0.334
	F <sub>sup</sub>	0.612	0.767	0.275	0.408
	L <sub>c</sub>	0.553	0.705	0.221	0.381
<b>LLE</b>	F <sub>mean</sub>	0.738	0.883	0.334	0.414
	F <sub>sup</sub>	0.720	0.867	0.309	0.407
	L <sub>c</sub>	0.651	0.821	0.287	0.346

## 4 Conclusion

In this paper we have considered alternative estimation methods of cointegrated vectors. The estimators are asymptotically efficient and allow us to conduct inference using standard asymptotic chi-squared tests. We use these estimators to compute structural change tests with unknown break points in cointegrated relationships. Our simulation results show that these tests work well in practice with sample sizes equal to 100, 200, and 500, leading to little size distortion and having the ability to detect structural change for a wide range of the simple break point alternatives.

All the tests exhibit slightly higher power with larger samples, but, interestingly, the power of the tests deteriorates substantially with increased error serial correlation. The  $F_{\text{mean}}$  tests based on the leads and lags estimator have overall better power than the competing tests when the sample size is large. With smaller-size data, we found that the FME based tests, in particular the  $F_{\text{sup}}$ , outperform the other competing tests.

Furthermore, our simulation experiments show that CCR-based tests are generally less powerful than LLE and FME based tests. One possible explanation is that when we estimate the cointegrated vector using canonical cointegration transformations we use an initial estimator which is not efficient.

We cautiously recommend the structural change tests analyzed in this paper, since they show an overall good performance in finite samples, although it has been found that they are not robust to strong serial correlation.

As a direction for future research, it will be of considerable theoretical and practical importance to assess the finite-sample performance of these tests with multiple cointegrated vectors.

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